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Econometrics of co-jumps in high-frequency data with noise

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Abstract

We establish estimation methods to determine co-jumps in multivariate high-frequency data with non-synchronous observations and market microstructure noise. The ex-post quadratic covariation of the signal part, which is modeled by an Itô-semimartingale, is estimated with a locally adaptive spectral approach. Locally adaptive thresholding allows to disentangle the co-jump and continuous part in quadratic covariation. Our estimation procedure implicitly renders spot (co-)variance estimators. We derive a feasible stable limit theorem for a truncated spectral estimator of integrated covariance. A test for common jumps is obtained with a wild bootstrap strategy. We give an explicit guideline how to implement the method and test the algorithm in Monte Carlo simulations. An empirical application to intra-day tick-data demonstrates the practical value of the approach.

Keywords: co-jumps, covolatility estimation, jump detection, microstructure noise,
non-synchronous observations, quadratic covariation, spectral estimation, truncation
JEL classification: C14, G32, E58

1. Introduction

Last years have seen a tremendous increase in intra-day trading activities. High-frequent trading stimulated a new angle on financial modeling arousing great interest in the field of statistics of ultra high-frequency data (UHF-data). Asset prices recorded as UHF-data are almost close to continuous-time observations and thus foster statistical inference for continuous-time price models. Demanding absence of arbitrage leads to models in which asset prices are described by semimartingales, see Delbaen and Schachermayer (1994), Imkeller and Perkowski (2013) and references therein. These include recent price models allowing for stochastic volatility and leverage. Though there is an ongoing discussion if log-prices can be more accurately modeled by pure jump-type or continuous semimartingales, there is a broad consensus that (large) jumps occur as responses to news flow in the markets.

Our main focus is on relevant news that affect various markets and assets simultaneously and may come from policy announcements or macroeconomic data releases. We detect such co-jumps from UHF-data accounting for market microstructure and non-synchronous trading. For portfolio and risk

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management, it is essential to understand (co-)jumps dynamics in UHF-data to distinguish between idiosyncratic and systemic risk. The presented approach provides access to study concerted or distinct reactions of different assets to events by quantifying and locating co-jumps. This is of key interest in various applications, e. g. to study default contagion. To this end, we present a locally adaptive spectral approach to draw statistical inference on the quadratic covariation of a multi-dimensional Itô-semimartingale from discrete UHF-data. Our method allows to separately estimate co-jumps and integrated covariance (sometimes called integrated covolatility), both disentangled from microstructure frictions, in an efficient way. It relies on a convenient combination of the spectral estimator by Bibinger and Reiß (2013) to cope with noise and truncation methods in the vein of Mancini (2009) and Jacod (2008)

In the one-dimensional case various estimation methods for the integrated volatility from discretely observed semimartingales with jumps have been developed. In this context, let us mention the important contributions by Barndorff-Nielsen and Shephard (2006), Jiang and Oomen (2008), Bollerslev et al. (2008), Mancini (2009), Jacod (2008), Fan and Wang (2007), Podolskji and Ziggel (2010) and Curci and Corsi (2012). Aït-Sahalia and Jacod (2009) have established a test for the presence of jumps. An overview and an empirical comparison is given in Theodosiou and Zikes (2011). In contrast to the one-dimensional case, there is scant literature on the multivariate setup yet. An important step for considering co-jumps in a multi-dimensional framework and extending truncation methods has been laid by Jacod and Todorov (2009) and Gobbi and Mancini (2012). However, their estimators are designed for non-perturbed observations.

One main contribution of this article is to develop a tractable estimator for more complex models taking market microstructure into account. Under noise perturbation the identification and localization of (co-)jumps is more challenging, since the principle that large returns represent (large) jumps in the efficient log-price is not valid due to the impact of microstructure. Inference on the volatility of a continuous semimartingale under noise contamination can be pursued using smoothing techniques. Several approaches have been invented, prominent ones by Zhang (2006), Barndorff-Nielsen et al. (2008), Jacod et al. (2009) and Xiu (2010) in the one-dimensional setting and generalizations for a noisy non-synchronous multi-dimensional setting by Aït-Sahalia et al. (2010), Barndorff-Nielsen et al. (2011), Park and Linton (2012), Christensen et al. (2011) and Bibinger and Reiß (2013), among others. A recent advance towards the estimation of the integrated covariance of a semimartingale with jumps has been given in Jing et al. (2012). In contrast to the majority of previous approaches, our main focus is on estimating co-jumps instead of concentrating only on the continuous part of quadratic covaration.

One building block for our approach is the spectral estimator by Bibinger and Reiß (2013). It relies on a locally quasi-parametric estimation technique in the Fourier domain. Structural results about the information content inherent in the statistical experiments by Bibinger et al. (2013) show that it can even attain the minimum asymptotic variance. Non-synchronicity is proved to be asymptotically negligible in combination with noise at the slower optimal convergence rate. In the light of these findings, and differently to preceding methods as Christensen et al. (2011) and Jing et al. (2012), we construct our estimators with equispaced blocks equally for all components to average noisy observations. This reduces the estimator's variance. Moreover, in the presence of co-jumps this attribute, that no interpolations are pursued to deal with non-synchronicity, is at the same time advantageous, since returns with jumps can not be considered more than once which could crucially complicate a co-jump estimator's distribution. An approach combining the spectral estimator and block-wise truncation provides an estimator for integrated covariances in the presence of (co-)jumps. Consequently, we estimate co-jumps by taking the difference of the non-truncated and truncated estimator. We obtain a feasible central limit theorem for the truncated estimator allowing for confidence. Finally, a locally adaptive thresholding strategy involving pre-estimated spot covariances renders an effective finite-sample approach. Furthermore, a co-jump localization procedure in the spirit of Lee and Mykland (2008) is feasible. In order to derive a test for the presence of co-jumps, we adopt a strategy related to the wild bootstrap principle by Wu (1986), compare Podolskji and Ziggel (2010) who have constructed a test for jumps of one-dimensional semimartingales.

The article is arranged in six sections. In the next section, we introduce the statistical model and fix the notation. Theoretical results are given in Section 3, where we also carry out the construction of the estimation approach. In Section 4, we pursue the asymptotic theory for the test for co-jumps based on the wild bootstrap idea. Section 5 comes up with an implementation of the econometric estimation procedure for UHF-data – adjusted to finite sample issues and discussing some practical features. In Section 6 we investigate our approach in a simulation study and show its applicability in an empirical example. Section 7 concludes. Technical proofs are postponed to the Appendix.

2. Theoretical setup

We consider prices recorded as UHF-data from d individual assets. The evolution of hypothetic underlying continuous-time log-price processes are driven by a d-dimensional Itô-semimartingale

$$\begin{aligned} X_t &= X_0 + \int_0^t b_s\,ds + \int_0^t \sigma_s\,dW_s + \int_0^t \int_{\mathbb{R}^d} \kappa(\delta(s,x))(\mu-\nu)(ds,dx) + \int_0^t \int_{\mathbb{R}^d} \bar{\kappa}(\delta(s,x))\mu(ds,dx) \\ &= C_t + J_t\,, t \in \mathbb{R}_+, \end{aligned} \tag{1}$$

on a suitable filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$ with a right-continuous and complete filtration. The first three addends are composed to the continuous part $(C_t)_{t\ge 0}$ with W being a d-dimensional standard Wiener process and σ_t is the stochastic instantaneous volatility process. The jump part $(J_t)_{t\ge 0}$ is decomposed in a finite sum of large jumps and compensated small jumps using a truncation function κ. The Poisson random measure μ on $(\mathbb{R}_+ \times \mathbb{R}^d)$ is compensated by its intensity measure $\nu(ds,dx) = ds \otimes \lambda(dx)$ with a σ-finite measure λ on $\mathbb{R}^d$ endowed with the Borelian σ-algebra. If $\lambda(\mathbb{R}^d) = \infty$ the process is said to have infinite activity. The truncation function $\kappa : \mathbb{R}^d \to \mathbb{R}^d$, $\bar{\kappa}(z) = z - \kappa(z)$, separates small and large jumps. A predictable function δ is used to shift the cut-off in time and alternatively a transition measure λ_s can be incorporated to write $\kappa(x)ds\lambda_s(dx)$ instead of $\kappa(\delta(s,x))\nu(ds,dx)$. Our notation follows Jacod (2012) and Jacod and Todorov (2009), among others, and we refer to Jacod (2012) for background information on semimartingales. The structural assumption is accomplished by the following restrictions on the characteristics of the semimartingale.

Assumption (H). *The drift is a d-dimensional $(\mathcal{F}_t)$-adapted locally bounded process, the volatility σ is a $d \times d'$-dimensional $(\mathcal{F}_t)$-adapted continuous Itô-semimartingale whose drift and volatility are locally bounded and $\sup_{\omega,s,x} \|\delta_\omega(s,x)\|/\gamma(x)$ is locally bounded for a deterministic non-negative function γ satisfying*

$$\int_{\mathbb{R}^d} (\gamma^r(x) \wedge 1)\lambda(dx) < \infty\,, \tag{2}$$

for $r = 2$, or in some case for specified $r \in (0,2]$, as stated below.

The smallest possible r such that (2) holds is sometimes called the generalized Blumenthal-Getoor index, also referred to as jump activity index of semimartingales.
The quadratic covariation of the semimartingale X is the sum of the integrated covariance matrix $\Sigma_s = \sigma_s\sigma_s^\top$ and the co-jumps:

$$[X,X]_T = \int_0^T \Sigma_t\,dt + \sum_{s\le T}(X_s - X_{s-})(X_s - X_{s-})^\top = [C,C]_T + [J,J]_T\,. \tag{3}$$

The co-jumps $[J,J]_T = \int_0^T \int_{\mathbb{R}^d} \delta(s,x)\delta(s,x)^\top \mu(ds,dx)$ are written in (3) as a sum of all common jumps, where $X_{s-} = \lim_{t\to s, t<s} X_t$. Quadratic covariation is of pivotal importance to quantify risk in financial economics and is the target of inference in this article. We consider a general discrete observation setup including non-synchronous sampling schemes and market microstructure.

Assumption 1. *A d-dimensional semimartingale X of the type* (1) *is discretely and non-synchronously observed on $[0,T]$ at observation times $t_i^{(p)}, 0 \le i \le n_p, p = 1,\ldots,d$. The observations are corrupted with additive microstructure noise:*

$$Y_i^{(p)} = X^{(p)}_{t_i^{(p)}} + \epsilon_i^{(p)}\,, 0 \le i \le n_p\,.$$

The microstructure noise is given as a discrete-time process, mutually independent for all components, for which the observation errors are assumed to be i. i. d. and independent of X. Furthermore, the errors are centered and fourth moments exist.

We write $\Delta_i Y^{(p)} = Y_i^{(p)} - Y_{i-1}^{(p)}, 1 \le i \le n_p, p = 1,\ldots,d$ for the increments of $Y^{(p)}$ and $\mathbb{V}\mathrm{ar}\big(\epsilon_i^{(p)}\big) = \eta_p^2, 0 \le i \le n_p, p = 1,\ldots,d$, for the variance of the observation errors. Denote the number of observations of the least frequently traded asset by $n_{min} = \min_p n_p$. Quantities depending on n_{min} or some n_p are often indexed with n in the sequel.

Assumption 2. *Suppose that there exist differentiable distribution functions $F_p : [0,T] \to [0,T], p = 1, \dots, d$, with $F_p(0) = 0$, $F_p(T) = 1$ and $F_p' > 0$, such that the sampling times in Assumption 1 are generated by the quantile transformations $t_i^{(p)} = F_p^{-1}(iT/n_p)$, $0 \le i \le n_p$, $p = 1, \dots, d$.*

We emphasize that we treat observation schemes which are deterministic or random and independent of the process Y. A theory embedding endogenous random sampling calls for further mathematical concepts, just developed for simpler models, see Fukasawa and Rosenbaum (2012) and references therein. Assumption 1 comprises standard assertions on the noise as in related literature. An extension to m-dependence and mixing errors can be attained similar as in Aït-Sahalia et al. (2011). For notational convenience, we restrict ourselves to an usual i. i. d. assumption. Since we shall concentrate on the non-synchronous setup, we simply assume the componentwise noise processes to be mutually independent. An extension to $\mathbb{E}\left[\epsilon_i^{(p)}\epsilon_v^{(q)}\right] = \eta_{pq}$ if $t_i^{(p)} = t_v^{(q)}$, at synchronous observation times is direct as for the synchronous framework in Bibinger and Reiß (2013).
We write $a_n \asymp b_n$ to express that $a_n = \mathcal{O}(b_n)$ and $b_n = \mathcal{O}(a_n)$ for deterministic, and $a_n = \mathcal{O}_p(b_n)$ and $b_n = \mathcal{O}_p(a_n)$ for random sequences. $\xrightarrow{\mathbb{P}}$ denotes convergence in probability on $(\Omega, \mathcal{F}, \mathbb{P})$; $\rightsquigarrow$ and $\overset{st}{\rightsquigarrow}$ denotes weak and $\mathcal{F}$-stable weak convergence, respectively. See Jacod and Protter (1998) for the notion of stable weak convergence..

3. Spectral estimation of co-jumps and (integrated) covariance

3.1. Construction and discussion of the estimators

In this section we develop the spectral co-jump estimator. It is based on the spectral covariance estimator by Bibinger and Reiß (2013) as one building block and truncation to disentangle co-jumps and the continuous part. We briefly recapitulate the spectral covariance estimation approach. Thereto consider an orthonormal system of specific sine functions with support on the blocks $[kh_nT, (k+1)h_nT], k = 0, \dots, h_n^{-1} - 1$, with $h_n^{-1} \in \mathbb{N}$, and spectral frequencies $j \ge 1$:

$$\Phi_{jk}(t) = \frac{\sqrt{2h_n}}{j\pi\sqrt{T}} \sin\left(j\pi h_n^{-1}T^{-1}(t - kh_nT)\right)\mathbb{1}_{[kh_nT,(k+1)h_nT]}(t),\ j \ge 1\,, k = 0, \dots, h_n^{-1} - 1\,. \quad (4)$$

The functions (4) are weight functions providing spectral statistics for each frequency j localized on h_n^{-1} blocks:

$$S_{jk}^{(p)} = \sum_{i=1}^{n_p} \Delta_i Y^{(p)} \Phi_{jk}\left(\frac{t_i^{(p)} + t_{i-1}^{(p)}}{2}\right)\ ,\ j \ge 1, p = 1, \dots, d, k = 0, \dots, h_n^{-1} - 1. \quad (5)$$

With weights $w_{jk}^{p,q} \ge 0$ satisfying $\sum_{j\ge1} w_{jk}^{p,q} = 1$ for all k, the oracle spectral estimator is defined as

$$\mathbf{SPECV}_{n,T}^{(p,q)}(Y) = \sum_{k=0}^{h_n^{-1}-1} h_n \sum_{j=1}^{J_n} w_{jk}^{p,q} \frac{\pi^2 j^2}{h_n^2}\left(S_{jk}^{(p)}S_{jk}^{(q)} - \frac{\delta_{p,q}\eta_p^2}{n_pF_p'(kh_nT)}\right) = \sum_{k=0}^{h_n^{-1}-1} \Delta_k\left[\widehat{X^{(p)}, X^{(q)}}\right], \quad (6)$$

with a spectral cut-off frequency $J_n \le nh_n$ for $(p,q) \in \{1,\dots,d\}^2$ and

$$\Delta_k\big[\widehat{X^{(p)},X^{(q)}}\big] = \sum_{j=1}^{J_n} w_{jk}^{p,q} \frac{\pi^2 j^2}{h_n}\left(S_{jk}^{(p)}S_{jk}^{(q)} - \frac{\delta_{p,q}\eta_p^2}{n_p F_p'(kh_nT)}\right), \tag{7}$$

with $\delta_{p,q}$ being Kronecker's delta, i. e. 1 if $p=q$ and 0 else. In the presence of co-jumps, we prove that the spectral estimator consistently estimates the entire quadratic covariation (3).

The spectral approach relies on the idea to design an estimator for a locally parametric model in which a continuous martingale is observed discretely with noise and the covolatility matrix is constant over small blocks. The estimator can be understood as a localized generalized method of moments. The weights $w_{jk}^{p,q}$ are thus specified by block-wise Fisher informations, which depend on Σ_{kh_nT}, minimizing the variance of the oracle estimator (6). Plugging-in the pre-estimated instantaneous $\Sigma_t, t\in[0,T]$, renders the final locally adaptive spectral estimator. Note that the standardization in (6) $\pi^2j^2h_n^{-2}$ slightly differs from the one in Bibinger and Reiß (2013) which is for equidistant synchronous observations whereas we consider non-synchronous sampling here. Local adaptivity constitutes one of the main merits compared to previous methods with globally fixed tuning parameters for smoothing, and makes the estimator more flexible and efficient for time-varying (co-)volatility processes.

We act in the following as if the noise variances $\eta_p^2, p=1,\dots,d$, were known what does not harm the generality, since we can always estimate noise variances with faster convergence rates $\sqrt{n_p}$ by

$$\widehat{(\eta_p^2)} = (2n_p)^{-1}\sum_{i=1}^{n_p}\left(\Delta_i Y^{(p)}\right)^2 \text{ or } \bar{(\eta_p^2)} = -n_p^{-1}\sum_{i=1}^{n_p-1}\Delta_i Y^{(p)}\Delta_{i+1}Y^{(p)}. \tag{8}$$

The estimator in (6) has originally been designed to estimate the integrated covariance of continuous (semi)martingales $X=C$, $J=0$, and estimates the total quadratic covariation in the presence of co-jumps. One way to separately estimate the integrated covariance matrix is a truncated spectral estimator:

$$\begin{aligned}\mathbf{TSPECV}_{n,T}^{(p,q)}(Y,u_n) &= \sum_{k=0}^{h_n^{-1}-1}\Delta_k\big[\widehat{X^{(p)},X^{(q)}}\big]\mathbb{1}_{\{|\Delta_k[\widehat{X^{(p)},X^{(q)}}]|\le u_n\}} \\ &= \sum_{k=0}^{h_n^{-1}-1}\left(h_n\sum_{j=1}^{J_n} w_{jk}^{p,q}\frac{\pi^2j^2}{h_n^2}\left(S_{jk}^{(p)}S_{jk}^{(q)} - \frac{\delta_{p,q}\eta_p^2}{n_pF_p'(kh_nT)}\right)\right)\mathbb{1}_{\{|\Delta_k[\widehat{X^{(p)},X^{(q)}}]|\le u_n\}},\end{aligned} \tag{9}$$

with a truncation cut-off $u_n = c\,h_n^{\tau}$, $\tau\in(0,1)$, $c>0$. Consequently, the spectral estimator of co-jumps is derived as the difference by the non-truncated and the truncated SPECV:

$$\begin{aligned}\mathbf{SPECJ}_{n,T}^{(p,q)}(Y,u_n) &= \mathbf{SPECV}_{n,T}^{(p,q)}(Y) - \mathbf{TSPECV}_{n,T}^{(p,q)}(Y,u_n) \\ &= \sum_{k=0}^{h_n^{-1}-1}\Delta_k[\widehat{X^{(p)},X^{(q)}}]\mathbb{1}_{\{|\Delta_k[\widehat{X^{(p)},X^{(q)}}]|>u_n\}}.\end{aligned} \tag{10}$$

In contrast to co-jumps, the continuous part contributes quadratic covariations of order h_n on blocks with shrinking lengths h_n. Therefore, we can truncate with a global threshold h_n^τ for some $\tau \in (0,1)$. For finite-sample applications it will be crucial to find a more sensitive thresholding rule pursued in Section 5 below.

For an adaptive fully data-driven estimator, we require pilot estimates of the oracle weights

$$w_{jk}^{p,q} = \frac{\Big(\frac{\pi^4 j^4 h_n^{-4}}{n_p n_q}\tilde{\eta}_p^2\tilde{\eta}_q^2 + \Big((\Sigma_{kh_nT}^{(pq)})^2 + \Sigma_{kh_nT}^{(pp)}\Sigma_{kh_nT}^{(qq)}\Big) + \frac{\pi^2 j^2 h_n^{-2}}{(n_p n_q)^{1/2}}\big(\Sigma_{kh_nT}^{(pp)}\tilde{\eta}_q^2 + \Sigma_{kh_nT}^{(qq)}\tilde{\eta}_p^2\big)\Big)^{-1}}{\sum_{r=1}^{J_n}\Big(\frac{\pi^4 r^4 h_n^{-4}}{n_p n_q}\tilde{\eta}_p^2\tilde{\eta}_q^2 + \Big((\Sigma_{kh_nT}^{(pq)})^2 + \Sigma_{kh_nT}^{(pp)}\Sigma_{kh_nT}^{(qq)}\Big) + \frac{\pi^2 r^2 h_n^{-2}}{(n_p n_q)^{1/2}}\big(\Sigma_{kh_nT}^{(pp)}\tilde{\eta}_q^2 + \Sigma_{kh_nT}^{(qq)}\tilde{\eta}_p^2\big)\Big)^{-1}}.$$

For $X = C, J = 0$, nonparametric estimates of the spot covariances $\Sigma_t^{(pq)}, t \in [0,T]$, can be obtained by local averages of the estimates $\Delta_k[\widehat{X^{(p)}, X^{(q)}}]$ which are approximately $h_n\Sigma_{kh_nT}^{(pq)}$. Bibinger and Reiß (2013) proposed to use only the first frequency, $j = 1$, and average over a set $\mathcal{K}_t$ of K_{pilot} adjacent blocks containing t. For semimartingales σ with $h_n \asymp n_{min}^{-1/2}$, $K_{pilot} \asymp n_{min}^{1/4}$, the root mean squared error is of order $\mathcal{O}(n_{min}^{-1/8})$. Uniform loss in t is bounded by (see Bibinger and Reiß (2013))

$$\mathbb{E}\left[\sup_{t\in[0,T]} |\hat{\Sigma}_t^{(pq)} - \Sigma_t^{(pq)}|\right] = \mathcal{O}\left((n_{min}/\log n_{min})^{-1/8}\right).$$

Truncation can eliminate jumps in the spot covariance estimator in the same way as for our truncated integrated covariance estimator. For this purpose we can use again a cut-off $u_n = c\, h_n^\tau$ with $\tau \in (0,1)$, $c > 0$, and estimate the spot covariance by

$$\hat{\Sigma}_t^{(pq)} = K_{pilot}^{-1}\sum_{k\in\mathcal{K}_t}\pi^2 h_n^{-2}\left(S_{1k}^{(p)}S_{1k}^{(q)} - \frac{\delta_{p,q}\eta_p^2}{n_p F_p'(kh_nT)}\right)\mathbb{1}_{\{|\Delta_k[\widehat{X^{(p)},X^{(q)}}]| \le u_n\}}. \tag{11}$$

The estimators using only one single frequency which are close to usual block-wise pre-averages can already attain optimal rates, yet in practice spot estimators involving more frequencies, which we introduce in Section 5 below achieved a higher efficiency. We plug in these piecewise constant estimates in the oracle weights together with the estimated rescaled local noise variances $\tilde{\eta}_p = \eta_p(F_p'(kh_nT))^{-1/2}$ with the given observation times distributions and (8). We keep to the same notation for the adaptive estimators as for the oracle ones and from this point only refer to the fully adaptive estimators.

The weights slightly differ from Bibinger and Reiß (2013) again, since we focus on a more general non-synchronous setup. Still, blocks are chosen equally along all asset processes which minimizes the estimator's variance. This is owed to a fundamental property of underlain statistical experiments that non-synchronicity is asymptotically immaterial in a combination with microstructure noise established in Bibinger et al. (2013).

3.2. Asymptotic properties of the estimators

In this section, we find consistency results for spectral estimators of integrated covariance, entire quadratic covariation and co-jumps. We establish a stable central limit theorem for the truncated estimator and integrated covariance and provide a feasible version allowing for confidence.

Theorem 1. *On Assumption (H) on the signal process X, Assumption 1 on the observation model and Assumption 2 on the observation times design, we derive the following asymptotic results for the estimators* (6), (9) *and* (10) *under high-frequency asymptotics as $n_{min} \to \infty, h_n \to 0$. The spectral estimator* (6) *is consistent for the quadratic covariation:*

$$\left(\mathbf{SPECV}_{n,T}^{(p,q)}(Y) - [X^{(p)}, X^{(q)}]_T\right) \xrightarrow{\mathbb{P}} 0\,. \tag{12a}$$

The truncated spectral estimator consistently estimates the integrated covariance (covolatility):

$$\left(\mathbf{TSPECV}_{n,T}^{(p,q)}(Y, u_n) - [C^{(p)}, C^{(q)}]_T\right) \xrightarrow{\mathbb{P}} 0\,, \tag{12b}$$

and the spectral co-jumps estimator consistently estimates the jump part of quadratic covariation:

$$\left(\mathbf{SPECJ}_{n,T}^{(p,q)}(Y, u_n) - [J^{(p)}, J^{(q)}]_T\right) \xrightarrow{\mathbb{P}} 0\,. \tag{12c}$$

Theorem 2. *If, additionally to the Assumptions of Theorem 1,* (2) *holds with $r < 1$, for $J_n \to \infty$ and $h_n \asymp \sqrt{n_p} \log n_p$, $n_p \asymp n_q$, $\tau > (2-r)^{-1}$, the following pairwise central limit theorem applies:*

$$n_p^{1/4}\left(\mathbf{TSPECV}_{n,T}^{(p,q)}(Y, u_n) - [C^{(p)}, C^{(q)}]_T\right) \stackrel{st}{\rightsquigarrow} \mathbf{N}\left(0, \mathbf{AVAR}\right)\,, \tag{13}$$

with the asymptotic variance (see Bibinger and Reiß (2013) for discussion)

$$\begin{aligned}\mathbf{AVAR} = &\int_0^T \sqrt{\tilde{\eta}_p(s)\tilde{\eta}_q(s)}\sqrt{2(A_s^2 - B_s)B_s} \\ &\times \left(\sqrt{A_s + \sqrt{A_s^2 - B_s}} - \operatorname{sgn}(A_s^2 - B_s)\sqrt{A_s - \sqrt{A_s^2 - B_s}}\right)^{-1} ds\,,\end{aligned} \tag{14}$$

$$A_s = \left(\Sigma_s^{(pp)} \frac{\tilde{\eta}_q(s)}{\tilde{\eta}_p(s)} + \Sigma_s^{(qq)} \frac{\tilde{\eta}_p(s)}{\tilde{\eta}_q(s)}\right)\,,\ B_s = 4(\Sigma_s^{(pp)}\Sigma_s^{(qq)} + (\Sigma_s^{(pq)})^2)\,,$$

where $\tilde{\eta}_p(s) = \eta_p/(F_p'(s))^{1/2}, \tilde{\eta}_q(s) = \eta_q/((F_q'(s))^{1/2}(n_q/n_p))$.

For the proof of consistency in Appendix A.2, we will establish the convergence

$$\left(\mathbf{SPECV}_{n,T}^{(p,q)}(C) - \mathbf{TSPECV}_{n,T}^{(p,q)}(C + J, u_n)\right) \xrightarrow{\mathbb{P}} 0\,, \tag{15}$$

stating that the truncation asymptotically eliminates the impact of jumps on the spectral covariation estimation and is thus eligible to estimate the two parts of quadratic covariation separately. If we can show on the assertion that jumps are of finite variation that the error in (15) is $\mathcal{O}_p(n_{min}^{-1/4})$, the central limit theorem (13) follows from the case without jumps and Bibinger (2013). We conjecture that for $\mathbf{SPECV}_{n,T}^{(p,q)}(Y)$ a central limit theorem holds with a second addend in the asymptotic variance due to a cross term by jumps and increments from the continuous part, but we do not focus on the exact asymptotic distribution of $\mathbf{SPECV}_{n,T}^{(p,q)}(Y)$ here, which requires a mathematically challenging analysis on its own.

It is known that the speed of convergence in (13) is optimal for integrated covariance estimation in a latent observation model. Moreover, the restriction to a jump component where (2) holds with $r < 1$ to prove a CLT (13) is natural, since the analogous assertion is already needed in Jacod (2008) for truncated realized variance in the simpler one-dimensional non-noisy observation experiment, where the rate is of course faster. For a discussion on optimality for this setup we refer to Jacod and Reiß (2012). Presumably, analogous reasoning with slower rate applies to the sequences of statistical experiments we consider. From our point of view this fact underlines the barriers in disentangling variation of small jumps and a continuous component. Yet, our focus is rather to separate co-jumps of larger magnitude and for this purpose truncation appears to be well-suited.

A feasible central limit theorem which affords confidence intervals is obtained implicitly, since the weights are determined via local Fisher informations $w_{jk}^{p,q} \propto I_{jk}^{p,q}$ which are block-wise inverse variances. A consistent estimator of the overall variance

$$\mathbb{V}\text{ar}\left(\mathbf{SPECV}_{n,T}^{(p,q)}(C)\right) = \sum_{k=0}^{h_n^{-1}-1} h_n^2 \left(\sum_{j=1}^{J_n} I_{jk}^{p,q}\right)^{-1}$$

is directly derived from the pre-estimated weights:

$$\min\left(n_p, n_q\right)^{-1/2} \widehat{\mathbf{AVAR}} = \sum_{k=0}^{h_n^{-1}-1} h_n^2 \left(\sum_{j=1}^{J_n} \hat{I}_{jk}^{p,q}\right)^{-1} .$$

Corollary 3.1. *On Assumptions (H), 1 and 2, if* (2) *holds with* $r < 1$, $\tau > (2-r)^{-1}$, *and for* $h_n \gtrsim \sqrt{n_{min}}$, *we have a feasible central limit theorem:*

$$\min\left(n_p, n_q\right)^{1/4} \left(\widehat{\mathbf{AVAR}}\right)^{-1/2} \left(\mathbf{TSPECV}_{n,T}^{(p,q)}(Y, u_n) - [C^{(p)}, C^{(q)}]_T\right) \rightsquigarrow \mathbf{N}\left(0,1\right) . \tag{16}$$

In particular, Corollary 3.1 is valid in a more general setting than Theorem 2, because we can allow for different speeds $n_p = \mathcal{O}(n_q)$, when standardizing with the estimated variance. At the same time, the feasible limit theorem (16) is most appealing to practitioners to draw confidence for the estimates from the normal distribution.

4. A bootstrap-type test for co-jumps

In Theorem 2 a central limit theorem for the truncated spectral integrated covariance estimator has been established which directly includes an asymptotic distribution free test for the hypothesis that the integrated covariance equals zero. A central limit theorem for the SPECJ which could render a test for the presence of co-jumps is not available which comes from the fact that under high-frequency asymptotics as $n_{min} \to \infty$, we can estimate co-jumps with asymptotically vanishing variance at rate $n_{min}^{1/4}$. One way to derive a test for finite-sample applications can be achieved adopting the wild bootstrap-type approach used by Podolskji and Ziggel (2010) for a similar testing problem in a one-dimensional setup. It is founded on the principle that if we disturb the addends of SPECJ by multiplication with suitable external independent random variables, we can approach an asymptotic distribution of the manipulated SPECJ which hinges on the distribution of those external random variables and the underlying process. Note that while in the one-dimensional framework the test by Podolskji and Ziggel (2010) is one possible testing procedure and others are available (e. g. by ratios of power variations, see Aït-Sahalia and Jacod (2009)), at least for the non-noisy case, alternative tests for our general multi-dimensional framework are not available to the best of our knowledge. Generalizations of tests as the one by Aït-Sahalia and Jacod (2009) are not obvious.

In the sequel we construct a test dedicated to the decision problem

$$\Omega_T^{no\,cj,p,q} = \{\omega | t \mapsto X_t(\omega) \text{ has no common jumps } (X_s^{(p)} - X_{s-}^{(p)})(X_s^{(q)} - X_{s-}^{(q)}) \neq 0 \text{ on } [0,T]\}$$

against the alternative

$$\Omega_T^{cj,p,q} = \{\omega | t \mapsto X_t(\omega) \text{ has co-jumps on } [0,T]\} \,.$$

For this purpose we define exogenous random variables $\zeta_k, k = 0, \ldots, h_n^{-1} - 1$, on a canonical orthogonal extension of $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$ denoted by $\left(\Omega^{\perp}, \mathcal{F}^{\perp}, \mathbb{P}^{\perp}\right)$. The test statistic, incorporating the structure of the SPECJ and the exogenous randomization is

$$T^n(Y) = \min(n_p, n_q)^{1/4} \sum_{k=0}^{h_n^{-1}-1} \Delta_k[\widehat{X^{(p)}, X^{(q)}}] \big(\mathbb{1}_{\{|\Delta_k[\widehat{X^{(p)}, X^{(q)}}]| > u_n\}} + \zeta_k \mathbb{1}_{\{|\Delta_k[\widehat{X^{(p)}, X^{(q)}}]| \le u_n\}}\big). \quad (17)$$

We employ i. i. d. random variables $\zeta_k, k = 0, \ldots, h_n^{-1} - 1$:

$$\mathbb{P}\left(\zeta_k = -\beta\right) = 0.5 = \mathbb{P}\left(\zeta_k = \beta\right) \,.$$

A useful rewriting of (17) with $\zeta_k = 1 - \tilde{\zeta}_k$, is:

$$T^n(Y) = \min\left(n_p, n_q\right)^{1/4} \sum_{k=0}^{h_n^{-1}-1} \Delta_k[\widehat{X^{(p)}, X^{(q)}}] \left(1 - \tilde{\zeta}_k \mathbb{1}_{\{|\Delta_k[\widehat{X^{(p)}, X^{(q)}}]| \le u_n\}}\right) \,. \quad (18)$$

We refer to Podolskji and Ziggel (2010) who use a related statistic as (18) for a discussion about the choice of randomization. It is crucial that the distribution is symmetric and the two-atomic nature makes the analysis simple. We will use $\beta = 0.1$ below in Section 6 for our applications.

Theorem 3. *For the test statistic* (17) *the central limit theorem*

$$T_{st}^n(Y) = T^n(Y)\left(\min\left(n_p, n_q\right)^{1/2} \sum_{k=0}^{h_n^{-1}-1}\left(\Delta_k[\widehat{X^{(p)}, X^{(q)}}]\right)^2 \mathbb{V}\mathrm{ar}^{\perp}\left(\zeta_k\right)\right)^{-1/2} \rightsquigarrow \mathbf{N}(0,1) \qquad (19)$$

applies on the hypothesis $\Omega^{no\, cj,p,q}$ *and* $T^n(Y) \stackrel{\mathbb{P}\otimes\mathbb{P}^{\perp}}{\longrightarrow} \infty$ *on* $\Omega^{cj,p,q}$*. Furthermore, when* $n_p \asymp n_q$*, a stable central limit theorem is valid*

$$T^n(Y) \stackrel{st}{\rightsquigarrow} \mathbf{N}\left(0, \mathbb{V}\mathrm{ar}^{\perp}\left(\zeta_k\right)\left(\mathbf{AVAR} + [X^{(p)}, X^{(q)}]_T^2\right)\right) \qquad (20)$$

with the asymptotic variance $\mathbf{AVAR}$ *from* (14) *above.*

From Theorem 3, we can deduce critical values or p-values by

$$\left(\mathbb{P}\otimes\mathbb{P}^{\perp}\right)_{\Omega_T^{no\, cj,p,q}}\left(T_{st}^n(Y) > q_{1-\alpha}\right) \to \alpha\ ;\ \left(\mathbb{P}\otimes\mathbb{P}^{\perp}\right)_{\Omega_T^{cj,p,q}}\left(T_{st}^n(Y) > q_{1-\alpha}\right) \to 1$$

for level $0 \le \alpha \le 1$ with the quantiles q_α of a standard normal distribution.

5. An econometric co-jump estimation and localization approach

The high-frequency asymptotic theory for the estimator (9) allows to plug in a constant threshold $u_n = c\, h_n^{\tau}$ for any $\tau \in (0,1)$ and constant $c > 0$ to filter out jumps in the path of X. Yet, a major task towards an applicable implementation is to set up an adequate finite-sample truncation rule. In the following we make use of the fact that the asymptotic magnitude of quadratic covariation from the continuous part is known, i. e. $\max_k |\Delta_k\left[C^{(p)}, C^{(q)}\right]| \asymp 2\log(h_n^{-1})\cdot h_n$. The vital point is that the increase of quadratic covariation locally hinges on the spot covariance. Therefore, we propose a locally adaptive truncation estimator:

$$\mathbf{TSPECV}_{n,T}^{(p,q)}(Y, u_n) = \sum_{k=0}^{h_n^{-1}-1} \Delta_k\left[\widehat{X^{(p)}, X^{(q)}}\right] \mathbb{1}_{\{|\Delta_k[\widehat{X^{(p)}, X^{(q)}}]| \le u_n(t)\}}, \qquad (21a)$$

$$\mathbf{SPECJ}_{n,T}^{(p,q)}(Y, u_n) = \sum_{k=0}^{h_n^{-1}-1} \Delta_k\left[\widehat{X^{(p)}, X^{(q)}}\right] \mathbb{1}_{\{|\Delta_k[\widehat{X^{(p)}, X^{(q)}}]| > u_n(t)\}}, \qquad (21b)$$

with a time-varying truncation cut-off function $u_n(t) = c_t\, h_n 2\log(h_n^{-1})$, and c_t will be chosen as absolute value of a data-driven spot covariance estimator. Motivated by the locally parametric ap-

- Set a priori threshold $u_n = 2\log K_n \cdot K_n^{-1}, K_n = h_n^{-1}$, and choose K_{pilot}, J_n. Calculate piecewise constant spot covariance estimator

$$\hat{\Sigma}^{(pq)}_{kh_nT} = K_{pilot}^{-1} \sum_{m\in\mathcal{K}_k} \sum_{j=1}^{J_n} J_n^{-1}\pi^2 j^2 h_n^{-2} \left(S_{jm}^{(p)} S_{jm}^{(q)} - \delta_{p,q} B_m^p\right) \mathbb{1}_{\{|\Delta_m[\widehat{X^{(p)},X^{(q)}}]|\le u_n\}}$$

where $B_m^p = (1/2h_n)\left(\sum_{v=1}^{n_p}(\Delta_v Y^{(p)})^2\right)\left(\sum_{mh_n\le t_v^{(p)}\le(m+1)h_n}(t_v^{(p)} - t_{v-1}^{(p)})^2\right)$,
$m = 0,\dots,h_n^{-1}-1, \mathcal{K}_k = \{k - \lfloor K_{pilot}/2\rfloor \wedge 0,\dots,k+\lfloor K_{pilot}/2\rfloor \vee (h_n^{-1}-1)\}$.

- Determine block-wise weights

$$\hat{I}_{jk}^{(pq)} = \left(\hat{\Sigma}^{(pq)}_{kh_nT}\right)^2 + \hat{\Sigma}^{(pp)}_{kh_nT}\hat{\Sigma}^{(qq)}_{kh_nT} + h_n^{-4}\pi^4 j^4 B_k^p B_k^q + h_n^{-2}\pi^2 j^2\left(B_k^p\hat{\Sigma}^{(qq)}_{kh_nT} + B_k^q\hat{\Sigma}^{(pp)}_{kh_nT}\right),$$

$$\hat{w}_{jk}^{(pq)} = \hat{I}_{jk}^{(pq)} / \left(\sum_{l=1}^{J_n}\hat{I}_{lk}^{(pq)}\right).$$

- Plug in estimated weights in estimators (9) and (10) with a block-wise varying threshold

$$u_n(k) = \left|\hat{\Sigma}^{(pq)}_{kh_nT}\right| K_n^{-1}\, 2\log K_n,\ 0\le k\le K_n\,.$$

Algorithm 1: Algorithm for the estimation procedure.

proximation as above, we build the spot estimator and locally adaptive threshold block-wise:

$$u_n(k) = \left|\hat{\Sigma}^{(pq)}_{kh_nT}\right| h_n 2\log(h_n^{-1}), k = 0,\dots,h_n^{-1}-1\,.$$

For local thresholding from noisy data, it will be crucial to incorporate local spot covariance estimates. Compared to locating jumps for each asset separately from the quadratic variation estimates, this also factors in correlations. The procedure now works as follows: We evaluate block-wise spectral statistics with constant weights and obtain a nonparametric spot estimator by local averaging and truncation. Then, we calculate the spectral estimators with adaptively chosen weights and locally adaptive threshold. For an illustration of the thresholding procedure see Figure 1 in the simulation study in Section 6 below. The method at the same time allows for sequential block-wise testing for co-jumps in the fashion as Lee and Mykland (2008) suggested for a one-dimensional non-noisy setup. However, differently as in the absence of noise, we can assign co-jumps only to certain blocks and not more exactly to observation time instants. Under noise, for particular increments we may not infer that there is a jump if the increment is large, since most large increments are induced by the microstructure. However, the noise is smoothed out when taking spectral statistics on blocks. Large products of block-statistics relate to co-jumps. Instead of using only the first frequency for spot estimators, it will be convenient to employ pilot estimators summing up spectral statistics with frequencies $1\le j\le J_{pilot,n}$ with some spectral cut-off frequency $J_{pilot,n} > 1$. Since we do not know local Fisher informations for the weights, we simply use equal weights $J_{pilot,n}^{-1}$ for the pilot estimator:

$$\hat{\Sigma}_t^{(pq)} = K_{pilot}^{-1} \sum_{k \in \mathcal{K}_t} \sum_{j=1}^{J_{pilot,n}} J_{pilot,n}^{-1} \pi^2 j^2 h_n^{-2} \Big(S_{jk}^{(p)} S_{jk}^{(q)} - \frac{\delta_{p,q} \eta_p^2}{n_p F_p'(kh_nT)} \Big) \mathbb{1}_{\{|\Delta_k [\widehat{X^{(p)}, X^{(q)}}]| \le u_n\}} . \tag{22}$$

The truncation cut-off is $u_n = 2\log(h_n^{-1})\, h_n$. Crucial tuning parameters are the bin-widths h_n and K_{pilot} for smoothing. The spectral cut-off can be chosen of order $J_{pilot,n} = J_n \asymp \log n$, with a constant of proportionality which we have chosen in practice between 3 and 10. By the growth behavior

$$w_{jk} \asymp \begin{cases} \big(h_n \sqrt{n_{min}}\big)^{-1} & \text{if } j \lesssim h\sqrt{n_{min}} \\ \big(h_n \sqrt{n_{min}}\big)^{3} j^{-4} & \text{if } j \gtrsim h\sqrt{n_{min}} \end{cases}$$

of the weights, we find this to be an accurate choice and also if J_n is not too large that constant weights $J_{pilot,n}^{-1}$ for the pilot estimator are suitable. The exact number of blocks h_n^{-1} within a reasonable range will not affect the estimates much and the estimator is quite robust to different choices. We emphasize that the bias of the local parametric approximation does not depend at all on h_n and the number of blocks $K = h_n^{-1} \in \mathbb{N}$. We recommend to use $30 \le K \le 100$ for daily estimation of UHF-data. In principle it is also possible to take different block lengths adapted to the volatility paths, e. g. equispaced in tick-time for $d = 1$ or refresh times for $d > 1$, but the differences have been rather small in our applications.
We summarize the algorithm for the implementation of the estimation procedure concisely in Algorithm 1.

6. Simulation study and an empirical example

In this section, we apply the implemented estimation procedure to several scenarios and suitable model specifications to reveal its finite-sample properties and accuracy. In each scenario we analyze spectral estimators, but accentuate different aspects. We highlight the estimators' applicability in an empirical data example using UHF-data from stock and bond markets. For the simulation study it is informative to restrict to the two-dimensional case, $d = 2$, and $T = 1$. We begin with a concise description of the implemented scenarios.

6.1. Description of scenarios

- **Scenario 1**: A completely parametric setup where

$$\Sigma = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix},$$

  is constant and with a constant drift $b = 0.1$ and a fixed number of jumps permits to track the estimators' quality in a simple setting. $X^{(1)}$ and $X^{(2)}$ are observed at times $t_i^{(p)}, 0 \le i \le$

$30000, p = 1, 2$, where $t_i^{(p)}$ are order statistics of uniformly drawn points on $[0, 1]$. The signal is corrupted by i. i. d. noise which is in both components normally centered distributed with standard deviation $\eta = 0.001$. For the jump part we simulate two common jumps and one separate jump in $X^{(1)}$ and $X^{(2)}$, respectively. Jump times are uniformly distributed on $[0, 1]$, and jump heights are normally distributed with expectation 1 and standard deviation 0.5.

- **Scenario 2**: Our main focus is on the time-varying nonparametric case. For an example of deterministic volatility functions, set

$$
\begin{aligned}
\sigma_t^{(1)} &= 0.1 - 0.08 \cdot \sin(\pi t),\ t \in [0, 1]\,,\\
\sigma_t^{(2)} &= 0.15 - 0.07 \cdot \sin((6/7) \cdot \pi t),\ t \in [0, 1]\,,\\
\rho_t &= 0.5 + 0.01 \cdot \sin(\pi t),\ t \in [0, 1]\,,
\end{aligned}
$$

where the volatilities are higher at opening and end of the observed interval and the correlation is rather persistent and only slowly varying, which mimics some basic realistic features. The noise is in both components i. i. d. normally centered distributed with $\eta = 0.001$. We add deterministic jumps to the continuous part, i. e. at times $t = 19/60, 49/60$ of $X^{(1)}$ with jump size L_1 and at time $t = 19/60$ of $X^{(2)}$ of size L_2. We implement synchronous equidistant sampling times $i/30000, 0 \leq i \leq 30000$ and the drift equals zero. Note that fixed jump arrival times violate the standard structural Assumption (H). Yet, it allows to investigate the localization accuracy of the method. The results are not affected by the specified chosen jump times above.

- **Scenario 3**: A realistic complex stochastic volatility model with serial dependence in noise and Poisson observation schemes. We add a coupled compound Poisson jump measure to simulate random jumps with normally distributed jump sizes.

The underlying observation times design is generated from a homogenous Poisson model with $\mathbb{E}[n_1] = \mathbb{E}[n_2] = 30000$, using the times generated on $[0, 1]$. The Poisson sampling is independent from the process Y.

The stochastic volatility model emulates the one by Barndorff-Nielsen et al. (2011). Hence, the signal part of simulated log-prices follow a bivariate factor model

$$
dX_t^{(p)} = \mu^{(p)}\, dt + \rho^{(p)}\, \sigma_t^{(p)} dB_t^{(p)} + \sqrt{(1 - (\rho^{(p)})^2)}\, \sigma_t^{(p)}\, dW_t^{(p)}, \quad p = 1, 2, \tag{23}
$$

where $B_t = (B_t^{(1)}, B_t^{(2)})^\top$ is a two-dimensional standard Brownian motion and W_t is a two-dimensional standard Brownian Motion independent of B_t. The spot variance functions follow Ornstein-Uhlenbeck processes

$$
\begin{aligned}
\sigma_t^{(p)} &= \exp\left(\beta_0^{(p)} + \beta_1^{(p)}\, \psi_t^{(p)}\right) \qquad \text{where} \\
d\psi_t^{(p)} &= \alpha^{(p)}\, \psi_t^{(p)}\, dt + dB_t^{(p)}, \qquad p = 1, 2.
\end{aligned} \tag{24}
$$

Accordingly, the leverage between $X_t^{(p)}$ and $\psi_t^{(p)}$ is $\rho^{(p)}$ and the correlation between $X_t^{(1)}$ and $X_t^{(2)}$ is constant $\sqrt{(1-(\rho^1)^2)\,(1-(\rho^2)^2)}$. In the following we use equal parameters $\mu = \mu^{(1)} = \mu^{(2)}$ for both components, analogously for α, β_0, β_1. We normalize as in Barndorff-Nielsen et al. (2011), such that $\beta_0 = \beta_1^2/\,(2\alpha)$ which implies $\mathbb{E}\left[\int_0^1 (\sigma_s^{(p)})^2\,ds\right] = 1$. We use an Euler discretization scheme and for the OU processes $\psi_t^{(l)}$ we exploit the fact that an exact discretization is available. The starting values of $\psi_t^{(p)}, p = 1,2$, are generated from its stationary distribution, i.e. $\psi_0^{(p)} \sim \mathbf{N}\left(0, (-2\alpha)^{-1}\right), p = 1,2$.

We disturb the signal process X by non-i. i. d. microstructure noise generated from a martingale difference model,

$$Y_j^{(p)} = X_{t_j^{(p)}}^{(p)} + \epsilon_j^{(p)}, \quad j = 1,\ldots,n_p,\, p = 1,2,$$

where the noise is conditionally on the signal distributed according to

$$\epsilon_j^{(p)}\Big|\left\{X_{t_j^{(p)}}^{(p)}, \sigma_{t_j^{(p)}}^{(p)}\right\} \sim \mathbf{N}\left(\gamma\epsilon_{j-1}^{(p)}, (\omega_j^{(p)})^2\right) \quad \text{with} \quad (\omega_j^{(p)})^2 = \xi^2\sqrt{\frac{1}{n_p}\sum_{i=1}^{n_p}(\sigma_i^{(p)})^4},$$

where ξ gives the so-called signal-to-noise ratio. This implies that the noise variance increases with the volatility of the signal (see Bandi and Russell (2006), among others). We fix the parameter configuration $\mu = 0.03, \beta_0 = -5/16, \beta_1 = 1/8, \alpha = -1/40$ as in Barndorff-Nielsen et al. (2011) and $\gamma = 0.1$ and $\xi = 0.005$. The jumps are generated by a Poisson process with two expected common jumps and two expected idiosyncratic jumps in each component. The jumps sizes are normally distributed with expectation and standard deviation parameter $\Lambda/10$.

- **Data example**: We investigate comovements of UHF-data on German stocks index futures (FDAX) and futures on 10 year German Government bonds (FGBL, price notation). The data is provided by the derivatives trading platform EUREX. We analyze 252 trading days from January 2008 to December 2008, which represents a crucial period of the global financial crisis. The focus is on trading hours from 9:00 to 17:00 CET resulting in approx. 10000 to 30000 ticks per day. For each future we filter the data with the same most frequently traded maturity (typically next ahead). Observation times are non-synchronous and the stylized facts of the data indicate microstructure effects. As commonly known for most financial data there are also features which are not perfectly accordant to the additive noise model (as zero returns).

In any example we implement the adaptive spectral estimators according to Algorithm 1 and do not use oracle weights in the simulations. We use piecewise constant spectral pilot estimators for Σ employing constant weights J_{pilot}^{-1}. For the simulations, we set $J_{pilot} = J = 35$, use 30 blocks, smooth over $K = 7$ adjacent blocks for the spot estimators and run 1000 Monte Carlo iterations. For

Table 1: Simulation results of Scenario 1.

$\mathbf{SPECJ}^{(1,2)} = 2.04 \pm 1.11$	$\mathbf{TSPECV}^{(1,2)} = 0.50 \pm 0.11$
$\mathbf{SPECJ}^{(1,1)} = 3.83 \pm 1.95$	$\mathbf{TSPECV}^{(1,1)} = 0.98 \pm 0.13$
$\mathbf{SPECJ}^{(2,2)} = 3.85 \pm 1.93$	$\mathbf{TSPECV}^{(2,2)} = 0.98 \pm 0.13$

Note. Rounded sample means $\pm$ standard deviations from 1000 MC iterations.

the data example we set $K = 24$, the spectral cut-off $J = 35$ and utilize the same adaptive truncation rule as for our simulations.

6.2. Results

6.2.1. Results and interpretation of Scenario 1

In order to approve the eligibility of our spectral co-jumps estimator to estimate the jump part in quadratic covariation, we compare estimates with the theoretically expected values in this simple setup with fixed numbers of jumps in each component and a parametric covariance matrix. For the evaluation of the fluctuation of jump heights by the $\mathbf{N}(1, 1/4)$-distribution, note that for $X = 1 + Z$ with $Z \sim \mathbf{N}(0, 1/4)$, we have

$$\mathbb{E}\left[X^2\right] = \mathbb{E}\left[(1+Z)^2\right] = 1 + \mathbb{E}\left[Z^2\right] = 5/4\,,$$

$$\mathbb{V}\text{ar}\left(X^2\right) = \mathbb{V}\text{ar}\left(1 + 2 \cdot Z + Z^2\right) = 4 \cdot \mathbb{V}\text{ar}(Z) + \mathbb{V}\text{ar}(Z^2) = 4 \cdot 1/4 + 2/16 = 9/8\,.$$

Thus, $\mathbb{E}[J^{(1)}, J^{(2)}]_T = 2$ and $\mathbb{E}[J^{(1)}, J^{(1)}]_T = \mathbb{E}[J^{(2)}, J^{(2)}]_T = 3.75$. The quadratic covariation of randomly generated jumps has a theoretical standard deviation of ca. 1.06 and the variations of ca. 1.84, respectively, and we expect the simulated ones to be slightly larger. Table 1 lists the Monte Carlo results which are quite close to their theoretical counterparts. For the truncated integrated covariance estimator, the finite sample variance under non-synchronous sampling is slightly larger than suggested from (14).

6.2.2. Results and interpretation of Scenario 2

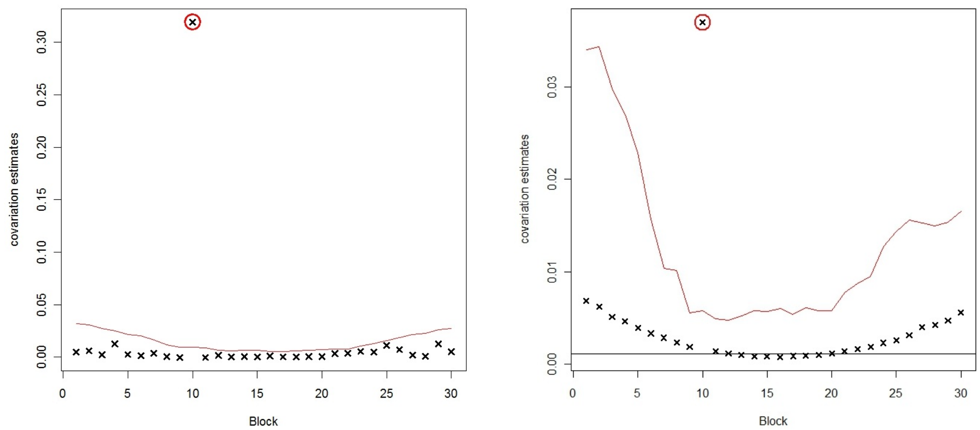
The known integrated covariance equals 0.00269 here and integrated variances 0.00301 and 0.01048, respectively. We consider four configurations:

M1 $L_1 = 0.1$ and $L_2 = 0.1$, where jumps are very large compared to Brownian increments (more than 100 times larger on average).

M2 $L_1 = 0.05$ and $L_2 = 0.1$.

M3 $L_1 = 0.05$ and $L_2 = 0.05$.

Figure 1: Block-wise estimated $\Delta_k\widehat{[X^{(1)},X^{(2)}]}$ in Scenario 2, setup M1 (left) and M4 (right).

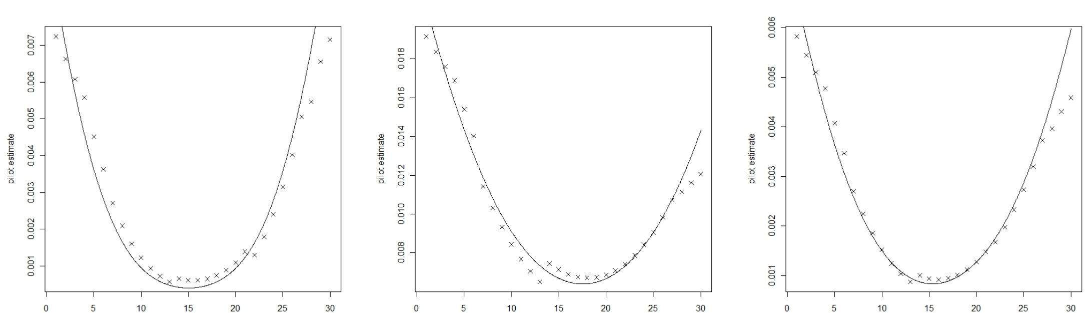


Note. Thresholding for $L_1 = 0.1$ (left) and $L_1 = 0.01$ (right). Marked by a circle are spectral statistics which exceed the threshold and are ascribed to a co-jump.

M4 $L_1 = 0.01$ and $L_2 = 0.1$, where the jumps of $X^{(1)}$ are not of much larger magnitude as increments from the continuous part, but still for $X^{(2)}$.

It is particularly interesting how the pilot estimates can mimic the paths of the spot variances and the covariance. The pilot estimators for M1 are illustrated in Figure 2. The performance of the spot estimators is quite remarkable and can mimic the functions satisfactorily, not only for the MC-averages but in any iteration. Figure 1 visualizes the estimation procedure of the spectral estimators (9) and (10). For each block the estimated increase of quadratic covariation (in a local parametric model simply $h_n\Sigma_{kh_nT}$) is componentwise compared to the local threshold $|\hat{\Sigma}_{kh_nT}|h_n\,2\log(h_n^{-1})$. We give an overview on the estimation results of the spectral estimators in Table 2. We also investigate the wild

Figure 2: Estimated and true spot (co-)variances in Scenario 2.



Note. Block-wise MC-averages. Spot variances of $X^{(1)}$ and $X^{(2)}$ (left, middle) and covariance (right).

Table 2: Simulation results for Scenario 2.

	M1	M2	M3	M4
$\mathbf{SPECJ}^{(1,2)}$	107 ± 4.5	53 ± 3.6	26 ± 6.5	8 ± 7.0
$\mathbf{SPECJ}^{(1,1)}$	206 ± 4.5	51 ± 3.9	51 ± 4.0	10 ± 5.2
$\mathbf{SPECJ}^{(2,2)}$	106 ± 6.4	106 ± 6.1	26.1 ± 5.9	106 ± 6.2
$\mathbf{TSPECV}^{(1,2)}$	26 ± 5.4	26 ± 4.5	27 ± 6.6	30 ± 6.2
$\mathbf{TSPECV}^{(1,1)}$	28.7 ± 2.7	31 ± 1.3	30 ± 3.2	32 ± 3.1
$\mathbf{TSPECV}^{(2,2)}$	101 ± 5.9	102 ± 5.9	105 ± 6.6	102 ± 5.7
T_{st}^n	29.5 ± 1.3	22.9 ± 1.3	14.8 ± 3.6	5.1 ± 4.6
1%-power	1.00	1.00	0.96	0.77
5%-power	1.00	1.00	0.96	0.77
10%-power	1.00	1.00	0.97	0.78
sensitivity	1.00	1.00	0.96	0.76

Note. Spectral estimators of quadratic (co)variation and (co)jumps (mean and standard deviation) times 10^4; test characteristics.

bootstrap test and list the results for the test statistics with empirical powers to certain test levels. The portion of detected co-jumps is given in the last row of Table 2. Concerning specificity we have 328, 80, 95 and 18 mistakenly detected co-jumps in the 1000 iterations in configuration M1, M2, M3 and M4, respectively. Most of them (319, 67, 69, 16) are located on block 25 (when one of both processes exhibits a jump).

6.2.3. Results and interpretation of Scenario 3

We implement the stochastic volatility model with expected jumps sizes $\Lambda = i/10, i = 0, \ldots, 10$. The estimators perform well in this realistic model which harms some of our technical assumptions on which theoretical asymptotic results are proved. In Figure 3 the power of the $\alpha = 0.01/0.05/0.1$-level co-jump test from Section 4 is depicted for fixed expected number of observations for different values of Λ. As the jumps sizes increase the power increases so long as jumps are very large compared to continuous part increments. Table 3 gives the root mean square errors (RMSE) of the SPECJ and the TSPECV. For increasing Λ (mean and standard deviation of jumps), the variance of SPECJ increases naturally. The RMSE of $\mathbf{TSPECV}^{(1,2)}$ slightly increases with Λ, whereas the RMSE for the one-dimensional estimators are slightly larger for moderate jump sizes. All truncated estimators exhibit the smallest RMSE in absence of jumps.
Altogether, simulation results are promising and we conclude that the spectral approach provides an efficient method to draw inference on co-jumps and quadratic covariation, even in complex models accounting for various dependencies.

6.2.4. Results and interpretation of data example

Figure 4 depicts SPECJ (black) and TSPECV (gray) estimates for each trading day. As economic intuition suggests, on most days the integrated covariance (gray) of bond and stock prices is negative. Interestingly, such a prevailing negative relation of common jumps (black) is not evident from Figure 4. The direction of co-jumps appears unsystematic. However, after applying the co-jump test, the

Table 3: Simulation RMSEs in Scenario 3.

Λ	$\mathbf{TSPECV}^{(1,2)}$	$\mathbf{TSPECV}^{(1,1)}$	$\mathbf{TSPECV}^{(2,2)}$	$\mathbf{SPECJ}^{(1,2)}$	$\mathbf{SPECJ}^{(1,1)}$	$\mathbf{SPECJ}^{(2,2)}$
10	0.052	0.032	0.033	1.42	1.35	1.41
9	0.049	0.030	0.031	1.16	1.11	1.14
8	0.048	0.033	0.033	0.93	0.79	0.91
7	0.044	0.037	0.035	0.66	0.73	0.53
6	0.041	0.036	0.033	0.51	0.45	0.54
5	0.040	0.040	0.032	0.33	0.32	0.33
4	0.039	0.049	0.052	0.21	0.18	0.21
3	0.039	0.060	0.063	0.12	0.12	0.13
2	0.036	0.060	0.063	0.07	0.10	0.11
1	0.033	0.062	0.064	0.06	0.06	0.03
0	0.023	0.024	0.025	0	0	0

Note. Root mean square errors of truncated spectral estimators (TSPECV) of quadratic (co)variation and (co-)jump estimators (SPECJ)

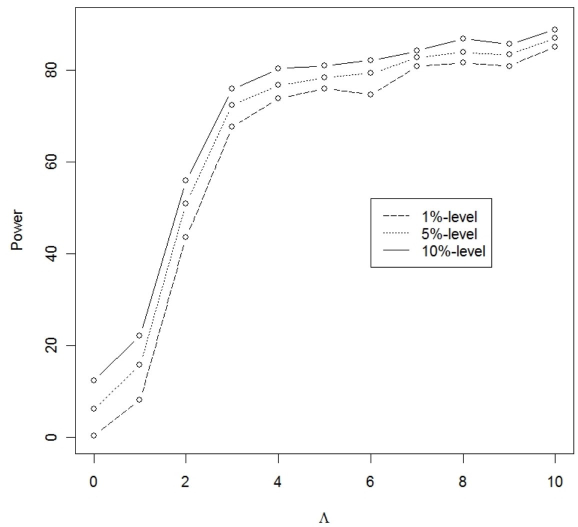
picture becomes clearer. On a 5% significance level, co-jumps occur on 19 days (7.5%), where only 3 days display positive co-jumps.

7. Conclusion

In this article we present a novel ex-post approach to test for and estimate co-jumps in multivariate high-frequency data which can cope with market microstructure and non-synchronous trading. Our estimation method brings together the efficient spectral covariance estimator by Reiß (2011) and Bibinger and Reiß (2013) and the concept of truncation. The spectral estimator attains substantial efficiency gains for time-varying volatility matrices compared to previous methods by its local adaptivity. We derive new estimators for the spot covariances and variances which we employ for choosing weights and the threshold locally on blocks. Block-wise estimates of quadratic covariation exceeding this threshold are ascribed to co-jumps. A feasible limit theorem for integrated covariance estimation is established.

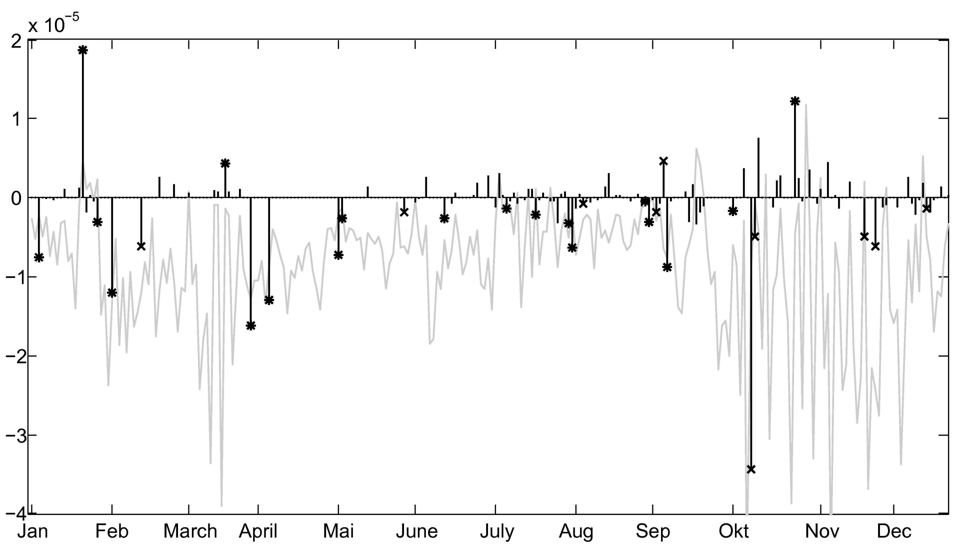
We investigate the performance of the invented estimation techniques in various simulation scenarios and an empirical example. The performance of our spot (co)variance estimators is remarkable and from our perspective they are worth further consideration in empirical work. Our approach paves the way for an efficient and rigorous analysis of the covariance structure and co-jump risks for applications in empirical finance. One important application we have in mind where inference on co-jumps takes a vital role is analyzing the impact of news announcements, see e. g. Lahaye et al. (2011). Our approach provides a capable and insightful tool which permits to expose co-jumps that appear near-term to events and quantifying their significance, magnitude and pairwise directions (concerted or contrast reaction).

Figure 3: Power of the test for the presence of co-jumps in Scenario 3.



Note. For fixed expected number of observation times power increases for larger jump heights.

Figure 4: Spectral estimates of co-jumps and covolatility for FDAX and FGBL.



Note. TSPECV (gray) and SPECJ (black) estimates for integrated covariance and co-jumps between FDAX and FGBL for trading days in 2008, $*$ and $\times$ mark significance on 5% and 10% level, respectively.

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Appendix A. Proofs

Appendix A.1. Some estimates and preliminaries

Throughout the proofs K denotes a generic constant and K_p denotes a generic constant dependent on some p. Considering a finite time span $[0, T]$, we can reinforce our structural Assumption (H) replacing local boundedness by uniform boundedness. This is standard in the strand of literature on statistics for semimartingales and based on a localization procedure provided in Jacod (2012), Lemma 6. 6 in Section 6. 3.

Assumption (SH). *We have Assumption (H) and, moreover, with some constant* Λ *for all* $(\omega, s, x) \in (\Omega, \mathbb{R}_+, \mathbb{R}^d)$*:*

$$\max\left\{\|b_s(\omega)\|, \|\sigma_s(\omega)\|, \|X_s(\omega)\|, \|\delta_\omega(s,x)\|/\gamma(x)\right\} \le \Lambda\,.$$

We decompose the semimartingale X which shall satisfy this structural Assumption in the continuous part C and jump part J and separate (co-)jumps with norm bounded from above by ε and larger jumps:

$$J_t = J(\varepsilon)_t + \int_0^t \int_{\mathbb{R}^d\setminus A_\varepsilon} \kappa(\delta(s,x))(\mu-\nu)(ds,dx) + \int_0^t \int_{\mathbb{R}^d} \bar{\kappa}(\delta(s,x))\mu(ds,dx)\,,$$

with $A_\varepsilon = \{z \in \mathbb{R}^d | \gamma(z) \le \varepsilon\}$ and later let $\varepsilon \to 0$. The strengthened boundedness assertion from Assumption (SH) is crucial for the following essential standard estimates which are used frequently in the sequel:

$$\forall p \ge 1, s, t \ge 0:\ \mathbb{E}\left[\|C_{s+t} - C_s\|^p \middle| \mathcal{F}_s\right] \le K_p t^{p/2}\,, \tag{A.1a}$$

$$\begin{aligned}\forall p \ge 1, \forall s, t \ge 0:\ \mathbb{E}\left[\|J(\varepsilon)_{s+t} - J(\varepsilon)_s\|^p \middle| \mathcal{F}_s\right] &\le K_p\,\mathbb{E}\Big[\Big(\int_s^{(s+t)} \int_{A_\varepsilon} (\gamma^2(z) \wedge 1)\mu(d\tau, dx)\Big)^{\frac{p}{2}}\Big] \\ &\le K_p t^{(\frac{p}{2}\wedge 1)} \gamma_\varepsilon^{(\frac{p}{2}\wedge 1)}\,, \end{aligned} \tag{A.1b}$$

$$\forall p \geq 1, s,t \geq 0: \ \mathbb{E}\left[\|C_{s+t}-C_s-(\sigma_s(W_{s+t}-W_s))\|^p \big| \mathcal{F}_s\right] \leq K_p\, \mathbb{E}\Big[\Big(\int_s^{s+t} \|\sigma_\tau - \sigma_s\|^2\, d\tau\Big)^{\frac{p}{2}} \big| \mathcal{F}_s\Big]$$
$$\leq K_p\, t^{\frac{p}{2}}\, \mathbb{E}\Big[\sup_{\tau \in [s,s+t]} (\|\sigma_\tau - \sigma_s\|^p) \big| \mathcal{F}_s\Big]$$
$$\leq K_p t^p\,, \tag{A.1c}$$

$$\forall s,t \geq 0: \ \mathbb{E}\left[\|J_{s+t} - J_s - (J(\varepsilon)_{s+t} - J(\varepsilon)_s)\| \big| \mathcal{F}_s\right] \leq K t \varepsilon^{-r}\,, \tag{A.1d}$$

with $r \in (0,2]$ from Assumption (H) in (A.1d) and $\gamma_\varepsilon = \int_{A_\varepsilon} \left(\gamma^2(x) \wedge 1\right) \lambda(dx)$. If we suppose (2), we have the bound

$$\gamma_\varepsilon \leq K \varepsilon^{(2-r)}\,. \tag{A.1e}$$

The estimates above are concluded by Burkholder-Davis-Gundy inequalities, Hölder inequality and Doob inequality and we refer to Jacod (2012), among others, for a detailed proof.

Appendix A.2. Proof of Theorems 1 and 2

The proof falls into three major parts, namely an approximation with a synchronous observation scheme, consistency of the SPECV for quadratic covariation and a bound for the difference of TSPECV(X) and SPECV(C). An essential ingredient of the analysis below is the result that we can work under a synchronous sampling design. For a continuous martingale observed in noise this fundamental result has been established in Bibinger et al. (2013) by virtue of an asymptotic equivalence in Le Cam's sense of the experiment with discrete non-synchronous observations and a continuous-time white noise experiment. Here, we show explicitly that on Assumptions 1 and 2, the approximation error for the signal part is asymptotically negligible. Note that the functions F_p', F_q' are considered in the noise part.

Lemma 1. *On Assumptions (H), 1 and 2, we can work under synchronous sampling, i. e. it holds that*

$$\sum_{k=0}^{h_n^{-1}-1} h_n \sum_{j=1}^{J_n} \pi^2 j^2 h_n^{-2} \sum_{v=1}^{n_p} \Delta_v X^{(p)} \Phi_{jk}(\bar{t}_v^{(p)}) \sum_{i=1}^{n_q} \Delta_i X^{(q)} \Phi_{jk}(\bar{t}_i^{(q)})$$
$$= \sum_{k=0}^{h_n^{-1}-1} h_n \sum_{j=1}^{J_n} \pi^2 j^2 h_n^{-2} \sum_{v=1}^{n_p} \left(X^{(p)}_{t_v^{(p)}} - X^{(p)}_{t_{v-1}^{(p)}}\right) \Phi_{jk}(\bar{t}_v^{(p)}) \sum_{v=1}^{n_p} \left(X^{(q)}_{t_v^{(p)}} - X^{(q)}_{t_{v-1}^{(p)}}\right) \Phi_{jk}(\bar{t}_v^{(p)}) + \mathcal{O}_p(1)\,.$$

Proof. Consider for $(p,q) \in \{1,\dots,d\}^2$, $\Delta_v t^{(p)} = t_v^{(p)} - t_{v-1}^{(p)}$, with the previous-tick and next-tick functions

$$t_+^{(p)}(s) = \min\left(t_v^{(p)}, 0 \leq v \leq n_p | t_v^{(p)} \geq s\right), p = 1, \dots, d, \tag{A.2a}$$

$$t_-^{(p)}(s) = \max\left(t_v^{(p)}, 0 \le v \le n_p | t_v^{(p)} \le s\right), p = 1, \ldots, d, \tag{A.2b}$$

and the mid-times

$$\bar{t}_v^{(p)} = \frac{t_{v-1}^{(p)} + t_v^{(p)}}{2}\,, 1 \le v \le n_p, p = 1, \ldots, d, \tag{A.2c}$$

the approximation

$$\begin{aligned}
\big(X^{(p)}_{t_v^{(p)}} - X^{(p)}_{t_{v-1}^{(p)}}\big)\Phi_{jk}(\bar{t}_v^{(p)}) &= \Big(\big(X^{(p)}_{t_v^{(p)}} - X^{(p)}_{t_-^{(q)}(t_v^{(p)})}\big) + \sum_{\Delta_i t^{(q)} \subset \Delta_v t^{(p)}} \big(X^{(p)}_{t_i^{(q)}} - X^{(p)}_{t_{i-1}^{(q)}}\big) \\
&\qquad + \big(X^{(p)}_{t_+^{(q)}(t_{v-1}^{(p)})} - X^{(p)}_{t_{v-1}^{(p)}}\big)\Big)\Phi_{jk}(\bar{t}_v^{(p)}) \\
\asymp \sum_{\Delta_i t^{(q)} \subset \Delta_v t^{(p)}} &\left(X^{(p)}_{t_i^{(q)}} - X^{(p)}_{t_{i-1}^{(q)}}\right)\Phi_{jk}(\bar{t}_i^{(q)}) + \big(X^{(p)}_{t_v^{(p)}} - X^{(p)}_{t_-^{(q)}(t_v^{(p)})}\big)\Phi_{jk}(u_v^{(q)}) \\
&\qquad + \big(X^{(p)}_{t_+^{(q)}(t_{v-1}^{(p)})} - X^{(p)}_{t_{v-1}^{(p)}}\big)\Phi_{jk}(\tilde{u}_v^{(q)})\,,
\end{aligned}$$

where $u_v^{(q)} = (1/2)(t_+^{(q)}(t_v^{(p)}) - t_-^{(q)}(t_v^{(p)}))$ and $\tilde{u}_v^{(q)} = (1/2)(t_+^{(q)}(t_{v-1}^{(p)}) - t_-^{(q)}(t_{v-1}^{(p)}))$. Since

$$\Phi_{jk}(t) - \Phi_{jk}(s) \asymp \Phi'_{jk}((t+s)/2)(t-s)$$

for small $(t-s)$, in particular the term is asymptotically at most of order $n_{min}^{-1/2}$ for $(t-s) \lesssim n_{min}^{-1}$. By the estimate with Φ'_{jk} above and (A.1a), (A.1b) and (A.1d), we can bound the approximation error

$$\begin{aligned}
&\sum_{k=0}^{h_n^{-1}-1} h_n \sum_{j=1}^{J_n} \pi^2 j^2 h_n^{-2} \sum_{v=1}^{n_p} \Delta_v X^{(p)} \Phi_{jk}(\bar{t}_v^{(p)}) \sum_{i=1}^{n_q} \Delta_i X^{(q)} \Phi_{jk}(\bar{t}_i^{(q)}) \\
&= \sum_{k=0}^{h_n^{-1}-1} h_n \sum_{j=1}^{J_n} \pi^2 j^2 h_n^{-2} \sum_{v=1}^{n_p} \left(X^{(p)}_{t_v^{(p)}} - X^{(p)}_{t_{v-1}^{(p)}}\right)\Phi_{jk}(\bar{t}_v^{(p)}) \sum_{v=1}^{n_p} \left(X^{(q)}_{t_v^{(p)}} - X^{(q)}_{t_{v-1}^{(p)}}\right)\Phi_{jk}(\bar{t}_v^{(p)}) \\
&+ \sum_{k=0}^{h_n^{-1}-1} \frac{\pi^2 j^2}{h_n} \sum_{j=1}^{J_n} \sum_{v=1}^{n_p} \Big(\big(X^{(p)}_{t_v^{(p)}} - X^{(p)}_{t_{v-1}^{(p)}}\big)\Big(\sum_{\Delta_i t^{(q)} \subset \Delta_v t^{(p)}} \big(X^{(q)}_{t_i^{(q)}} - X^{(q)}_{t_{i-1}^{(q)}}\big)\big(\Phi_{jk}(\bar{t}_i^{(q)}) - \Phi_{jk}(\bar{t}_v^{(p)})\big)\Big) \\
&+ \big(X^{(q)}_{t_+^{(q)}(t_{v-1}^{(p)})} - X^{(q)}_{t_{v-1}^{(p)}}\big)(\Phi_{jk}(\tilde{u}_v^{(q)}) - \Phi_{jk}(\bar{t}_v^{(p)})) + \big(X^{(q)}_{t_v^{(p)}} - X^{(q)}_{t_-^{(q)}(t_v^{(p)})}\big)\big(\Phi_{jk}(u_v^{(q)}) - \Phi_{jk}(\bar{t}_v^{(p)})\big)\Big)
\end{aligned}$$

in the last addend which is asymptotically negligible. □

Next, we establish the asymptotic properties of our estimators. We can restrict to a subset of Ω on which there is at most one large co-jump between two adjacent observation times, i. e. for

$$\mathcal{U}_{n,\varepsilon}^{i,(p,q)} = \Bigg\{\omega \big| \exists\, t \text{ such that } \left\| \left(\int_{t_{i-1}^{(p)}}^{t} \int_{A_\varepsilon^\complement} \delta(s,x)\, (\delta(s,x))^\top \mu(ds,dx) \right)^{(p,q)} \right\| > 0, \\ \left\| \left(\int_{t}^{t_i^{(p)}} \int_{A_\varepsilon^\complement} \delta(s,x)\, (\delta(s,x))^\top \mu(ds,dx) \right)^{(p,q)} \right\| > 0 \Bigg\}, 1 \le i \le n_p,$$

we can work on the complement of $(\bigcup_{i=1}^{n_p} \mathcal{U}_{n,\varepsilon}^{i,(p,q)}) \cup (\bigcup_{i=1}^{n_q} \mathcal{U}_{n,\varepsilon}^{i,(q,p)}) \to \emptyset$ as $n_{min} \to \infty$. The same reasoning can be further extended to blocks $[kh_nT, (k+1)h_nT)$, $k = 0, \dots, h_n^{-1}-1$, as $h_n \to 0$. Denote $\{S_1^{(p,q)}, \dots, S_{N(T)}^{(p,q)}\}$ the finite set of arrival times of large co-jumps[1], each one located on a different block. We augment this set for given n_{min} and fixed number of blocks h_n^{-1} to $\{S_1^{(p,q)}, \dots, S_{h_n^{-1}}^{(p,q)}\}$ by adding mid-times for blocks without large co-jump. With Lemma 1, we deduce

$$\begin{aligned}
&\mathbb{E}\left[\mathbf{SPECV}_{n,T}^{(p,q)}(Y)\right] = \sum_{k=0}^{h_n^{-1}-1} h_n \sum_{j=1}^{J_n} \pi^2 j^2 h_n^{-2} w_{jk}^{p,q} \left(\mathbb{E}\left[S_{jk}^{(p)} S_{jk}^{(q)}\right] - \frac{\delta_{p,q}\eta_p^2}{n_p F_p'(kh_nT)} \right) \\
&= \sum_{k=0}^{h_n^{-1}-1} h_n \sum_{j=1}^{J_n} \pi^2 j^2 h_n^{-2} w_{jk}^{p,q} \mathbb{E}\left[\sum_{i=1}^{n_p} \left(X_{t_i^{(p)}}^{(p)} - X_{t_{i-1}^{(p)}}^{(p)} \right) \Phi_{jk}(\bar{t}_i^{(p)}) \sum_{i=1}^{n_p} \left(X_{t_i^{(p)}}^{(q)} - X_{t_{i-1}^{(p)}}^{(q)} \right) \Phi_{jk}(\bar{t}_i^{(p)}) \right] + \mathcal{O}(1) \\
&= \sum_{k=0}^{h_n^{-1}-1} h_n \sum_{j=1}^{J_n} \pi^2 j^2 h_n^{-2} w_{jk}^{p,q} \sum_{i=1}^{n_p} (t_i^{(p)} - t_{i-1}^{(p)}) \Phi_{jk}^2(\bar{t}_i^{(p)}) \\
&\quad \times \left(\Sigma_{kh_nT}^{(pq)} + \left(\int_{\mathbb{R}^d \setminus A_\varepsilon} \delta(S_k^{(p,q)}, x) \left(\delta(S_k^{(p,q)}, x) \right)^\top \mu(S_k^{(p,q)}, dx) \right)^{(p,q)} \right) + \mathcal{O}(1) \\
&= \sum_{k=0}^{h_n^{-1}-1} h_n \left(\Sigma_{kh_nT}^{(pq)} + \left(\int_{\mathbb{R}^d} \delta(S_k^{(p,q)}, x)) \left(\delta(S_k^{(p,q)}, x) \right)^\top \mu(S_k^{(p,q)}, dx) \right)^{(p,q)} \right) + \mathcal{O}(1) \\
&\longrightarrow [X^{(p)}, X^{(q)}]_T \;\text{ as } \varepsilon \to 0\,.
\end{aligned}$$

By the bound (A.1b) and since $\gamma_\varepsilon \to 0$ for $\varepsilon \to 0$, the term by small jumps becomes asymptotically negligible. We have used that $\sum_i \Delta t_i \Phi_{jk}^2 \to \int_0^T \Phi_{jk}^2(t)\,dt = h_n^2 \pi^{-2} j^{-2}$. By standard bounds, the variance of SPECV is of order $n_{min}^{-1/2}$. Thus, we conclude that the SPECV is a consistent estimator for the quadratic covariation. Eventually, the proof that

$$\left(\mathbf{SPECV}_{n,T}^{(p,q)}(C) - \mathbf{TSPECV}_{n,T}^{(p,q)}(C+J, u_n) \right) = \mathcal{O}_P(1)\,,$$

and the stronger claim that the difference is $\mathcal{O}_P(n_{min}^{-1/4})$ on a reinforced assertion on the jump activity is

[1] Technically, this means we first consider co-jumps $\in A_\varepsilon^\complement$ and then letting $\varepsilon \to 0$, one may think of finitely many large co-jumps triggering the quadratic covariation here.

related to the strategy of proof pursued for truncated power variations in Aït-Sahalia and Jacod (2010). It relies on bounds for the increments from the continuous part, the small jumps and the probability to have co-jumps which exceed a certain threshold. Thereto, consider

$$\left(\mathbf{SPECV}_{n,T}^{(p,q)}(C)-\mathbf{TSPECV}_{n,T}^{(p,q)}(C+J,u_n)\right)=\sum_{k=0}^{h_n^{-1}-1}\zeta_k^{(p,q)}\,, \tag{A.3}$$

with $\zeta_k^{(p,q)}=\Delta_k[\widehat{C^{(p)},C^{(q)}}]$ in case of truncation, when $\Delta_k[\widehat{C^{(p)},C^{(q)}}]$ are the statistics (7) with latent observations of the continuous part C, and $-\Delta_k[\widehat{J^{(p)},J^{(q)}}]-\Delta_k[\widehat{C^{(p)},J^{(q)}}]-\Delta_k[\widehat{J^{(p)},C^{(q)}}]$ using analogous notation, else. We make a case differentiation:

- $\left|\Delta_k[\widehat{X^{(p)},X^{(q)}}]\right|>u_n$, truncation: $\zeta_k^{(p,q)}=\Delta_k[\widehat{C^{(p)},C^{(q)}}]$ and either $|\Delta_k[\widehat{C^{(p)},C^{(q)}}]|>u_n/2$ or $|\Delta_k[J^{(p)},J^{(q)}]|>u_n/2$, since the variance of $\Delta_k[\widehat{X^{(p)},X^{(q)}}]$ is of order h_n^2.
  - $\left|\Delta_k[\widehat{C^{(p)},C^{(q)}}]\right|>u_n/2$: For the truncation exponent $\tau\in(0,1)$, there exists an integer N_0 with $N_0(1-\tau)>1/2$ and

    $$\left|\zeta_k^{(p,q)}\right|\le 2^{N_0}\left|\Delta_k[\widehat{C^{(p)},C^{(q)}}]\right|\left(\frac{\left|\Delta_k[\widehat{C^{(p)},C^{(q)}}]\right|}{u_n}\right)^{N_0}\le 2^{N_0}\frac{\left|\Delta_k[C^{(p)},C^{(q)}]\right|^{N_0+1}}{u_n^{N_0}}$$
    $$=\mathcal{O}_p\left(h_n n_{min}^{-1/4}\right)$$

    by (A.1a).
  - $\left|\Delta_k[\widehat{C^{(p)},C^{(q)}}]\right|\le u_n/2$: Then $\left|\Delta_k[J^{(p)},J^{(q)}]\right|>u_n/2$ and hence

    $$\left|\zeta_k^{(p,q)}\right|\le 2^{N_0}\left|\Delta_k[\widehat{C^{(p)},C^{(q)}}]\right|\left(\frac{\left|\Delta_k[J^{(p)},J^{(q)}]\right|}{u_n}\right)^{N_0}=\mathcal{O}_p\left(h_n n_{min}^{-1/4}\right)$$

    what readily follows by (A.1b) and (A.1d).
- $\left|\Delta_k[\widehat{X^{(p)},X^{(q)}}]\right|\le u_n$, no truncation:
  - $\left|\Delta_k[\widehat{C^{(p)},C^{(q)}}]\right|>u_n/2$: With N_0 as above, we obtain that

    $$\left|\zeta_k^{(p,q)}\right|=\left|\Delta_k[\widehat{J^{(p)},J^{(q)}}]+\Delta_k[\widehat{C^{(p)},J^{(q)}}]+\Delta_k[\widehat{J^{(p)},C^{(q)}}]\right|$$
    $$\times 2^{N_0}\left(\frac{\left|\Delta_k[\widehat{C^{(p)},C^{(q)}}]\right|}{u_n}\right)^{N_0}=\mathcal{O}_p\left(h_n n_{min}^{-1/4}\right)\,.$$

  - $\left|\Delta_k[\widehat{C^{(p)},C^{(q)}}]\right|\le u_n/2$: Finally, we employ the estimates (A.1b), (A.1d) and (A.1e) to bound the absolute difference in the remaining case when there is no truncation and the continuous part is not of exceptionally large magnitude.

$$
\begin{aligned}
&\left|\Delta_k[\widehat{J^{(p)},J^{(q)}}]+\Delta_k[\widehat{C^{(p)},J^{(q)}}]+\Delta_k[\widehat{J^{(p)},C^{(q)}}]\right|\\
&\le\left(\left|\Delta_k[J^{(p)},J^{(q)}]\right|\wedge u_n\right)+\mathcal{O}_p\left(h_n n_{min}^{-1/4}\right)\asymp h_n u_n^{1-\frac{r}{2}}
\end{aligned}
$$

with the index r from Assumption (H). That the cross terms of continuous and jump part are asymptotically negligible is immediate by Cauchy-Schwarz and our usual estimates. The claim follows by applying (A.1b) and (A.1d) with $\varepsilon=u_n^{1/2}$. We need to ensure that

$$
\tau(1-\tfrac{r}{2})>1/2 \tag{A.4}
$$

and thus that $1>\tau>(2-r)^{-1}$ and $r<1$ to imply that $u_n^{1-r/2}=\mathcal{O}_p(n_{min}^{-1/4})$.

From the preceding analysis we obtain under the last restriction on the jump activity that

$$
\left(\mathbf{SPECV}_{n,T}^{(p,q)}(C)-\mathbf{TSPECV}_{n,T}^{(p,q)}(C+J,u_n)\right)=\sum_{k=0}^{h_n^{-1}-1}\zeta_k^{(p,q)}=\mathcal{O}_p\left(n_{min}^{-1/4}\right)\,. \tag{A.5}
$$

Appendix A.3. Proof of Theorem 3

Suppose the hypothesis $\Omega_T^{no\,cj,p,q}$ holds true (no co-jumps on $[0,T]$). Naturally, we have

$$
\mathcal{A}_{u_n}=\{\omega\in\Omega_T^{no\,cj,p,q}|\,|\Delta_k[\widehat{X^{(p)},X^{(q)}}]|\le u_n\ \forall k\}\to\Omega_T^{no\,cj,p,q}
$$

as $n_{min}\to\infty$ and $\mathcal{A}_{u_n}^{\complement}\to\emptyset$. By the typical standard argument we may in the sequel work on $\mathcal{A}_{u_n}$. Thus consider the conditional test statistic given Y:

$$
\tilde{T}^n(Y)=\min\left(n_p,n_q\right)^{1/4}\sum_{k=0}^{h^{-1}-1}\Delta_k[\widehat{X^{(p)},X^{(q)}}]\zeta_k\,.
$$

Conditional on the path of Y satisfying our assumptions (bounded moments), on $\left(\Omega^{\perp},\mathcal{F}^{\perp},\mathbb{P}^{\perp}\right)$ by the i. i. d. property of the $\zeta_k, 0\le k\le h_n^{-1}-1$, and

$$
\mathbb{E}^{\perp}\left[\tilde{T}^n(Y)|\mathcal{F}\right]=0\,,
$$

$$
\mathbb{V}\mathrm{ar}^{\perp}\left(\tilde{T}^n(Y)|\mathcal{F}\right)=\min\left(n_p,n_q\right)^{1/2}\sum_{k=0}^{h^{-1}-1}\left(\Delta_k[\widehat{X^{(p)},X^{(q)}}]\right)^2\mathbb{V}\mathrm{ar}^{\perp}\left(\zeta_k\right)\,,
$$

a central limit theorem is quickly derived by virtue of the fulfilled Lyapunov condition with higher moments. We now have conditionally on $\mathcal{F}$ that

$$\tilde{T}^n(Y)\left(\min\left(n_p,n_q\right)^{1/2}\sum_{k=0}^{h_n^{-1}-1}\left(\Delta_k[\widehat{X^{(p)},X^{(q)}}]\right)^2\mathbb{V}\mathrm{ar}^{\perp}\left(\zeta_k\right)\right)^{-1/2}\rightsquigarrow\mathbf{N}(0,1)\,.$$

On our original probability space $(\Omega,\mathcal{F},\mathbb{P})$ (here restricted to $\mathcal{A}_{u_n}$), we have convergence in probability of $\min\left(n_p,n_q\right)^{1/2}\sum_{k=0}^{h^{-1}-1}\left(\Delta_k[\widehat{X^{(p)},X^{(q)}}]\right)^2$. In particular, if $n_p\sim n_q$,

$$\min\left(n_p,n_q\right)^{1/2}\sum_{k=0}^{h_n^{-1}-1}\left(\Delta_k[\widehat{X^{(p)},X^{(q)}}]\right)^2\xrightarrow{\mathbb{P}}\mathbf{AVAR}+[X^{(p)},X^{(q)}]_T^2\,,$$

since the asymptotic variance of the truncated spectral estimator equals the one of $\mathrm{SPECV}_{n,T}^{(p,q)}(C)$ on $\Omega_T^{no\,cj,p,q}$ which has been deduced in Bibinger and Reiß (2013) and relies on the independence between blocks of the Brownian parts. The external nature of the $\zeta_k, 0\le k\le h_n^{-1}-1$, defined on an orthogonal extension $\left(\Omega^{\perp},\mathcal{F}^{\perp},\mathbb{P}^{\perp}\right)$ of $(\Omega,\mathcal{F},(\mathcal{F}_t),\mathbb{P})$, ensures that by Slutsky's lemma we obtain

$$T^n(Y)\left(\min\left(n_p,n_q\right)^{1/2}\sum_{k=0}^{h_n^{-1}-1}\left(\Delta_k[\widehat{X^{(p)},X^{(q)}}]\right)^2\mathbb{V}\mathrm{ar}^{\perp}\left(\zeta_k\right)\right)^{-1/2}\rightsquigarrow\mathbf{N}(0,1)\,. \tag{A.6}$$

We have exploited the stable central limit theorem for the spectral estimator on $\Omega_T^{no\,cj,p,q}$ here. We remark that the latter weak convergence is even stable with respect to $\mathcal{F}$ which implies the stable central limit theorem

$$T^n(Y)\overset{st}{\rightsquigarrow}\mathbf{N}\left(0,\mathbb{V}\mathrm{ar}^{\perp}\left(\zeta_k\right)\left(\mathbf{AVAR}+[X^{(p)},X^{(q)}]_T^2\right)\right)\,.$$

For the proof of the latter we refer to (65) and (66) on page 18 of Podolskji and Ziggel (2010), since it is along the same lines and the same argument applies as for their wild bootstrap-type statistics. Again, the external nature of the ζ_k plays a key role and implies stability.

Now suppose $\Omega_T^{cj,p,q}$. By virtue of Theorem 2, (12a), we have

$$\mathbf{SPECJ}_{n,T}^{(p,q)}\xrightarrow{\mathbb{P}}\sum_{s\le T}(X_s^{(p)}-X_{s-}^{(p)})(X_s^{(q)}-X_{s-}^{(q)})=[J^{(p)},J^{(q)}]_T\,.$$

Therefore,

$$\min\left(n_p,n_q\right)^{1/4}\sum_{k=0}^{h_n^{-1}-1}\Delta_k\left[\widehat{X^{(p)},X^{(q)}}\right]\mathbb{1}_{\left\{\left|\Delta_k\left[\widehat{X^{(p)},X^{(q)}}\right]\right|>u_n\right\}}=\mathcal{O}_p\left(\min\left(n_p,n_q\right)^{1/4}\right)\,, \tag{A.7}$$

and the same order applies to the first addend of the left-hand side of $T_{st}^{n}(Y)$ in (19). For the second addend of the left-hand side in (19), we infer for

$$\Gamma = \left(\sum_{k=0}^{h_n^{-1}-1}\left(\Delta_k\left[\widehat{X^{(p)},X^{(q)}}\right]\right)^2\mathbb{V}\mathrm{ar}^{\perp}(\zeta_k)\right)^{-1/2}\sum_{k=0}^{h_n^{-1}-1}\Delta_k\left[\widehat{X^{(p)},X^{(q)}}\right]\mathbb{1}_{\{|\Delta_k[\widehat{X^{(p)},X^{(q)}}]|\le u_n\}}\zeta_k$$

that $\mathbb{E}^{\perp}[\Gamma]=0$, $\mathbb{V}\mathrm{ar}^{\perp}(\Gamma)<1$. Thereby the assertion of Theorem 3 follows from (A.6) and (A.7).

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